AIAA JOURNAL
Vol. 31, No. 5, May 1993

Parameter-Transfer Finite Element Method for Structural Analysis

R. Barboni,* P. Gaudenzi,† and A. Mannini‡
Università di Roma "La Sapienza," Rome 00184, Italy

A theoretical formulation of a method to solve structural problems is given from a general point of view. The procedure creates a mathematical model of typical elements taking into account the whole behavior of the structure under concern such as static, dynamic, aerodynamic . . . thermal actions. In analogy with other numerical methods like finite elements, a "library" of models, named parameter-transfer finite elements (P-TFE) can be obtained for simple structures. Complex geometries can be approached by a discretization technique assembling the contributions from each element. An application to the panel flutter problem is given in which the proposed method overcomes most difficulties of the usual techniques of solution. To assess the validity of the method, a comparison with results obtained by other authors is made.

Nomenclature

$\{C\}_m$	= vector of integration constants
EI	= bending stiffness
$[I]$	= unit matrix
l	= length of the panel
M, $\bar{M}$	= bending moment
$[N]_m$	= shape function of mth order
$[N^*]$	= shape function at x_o
P	= axial load
q	= dynamic pressure
$[R]_i$	= transfer matrix of the ith element
T, $\bar{T}$	= shear force
w	= transverse displacement
x	= nondimensional independent variable
x_0	= value of x at the left of the element
$\{Y\}$	= vector of state variables
$\{Y^*\}$	= imposed conditions at x_0
$\{Y\}_m$	= vector of the generalized state variables
β	= $[(\text{Mach number})^2 - 1]^{1/2}$
γ	= normalized axial load
δ_{mn}	= Kronecker delta
ϑ	= rotation of the section of the plate
λ	= normalized eigenfrequency
μ	= mass density
σ	= normalized dynamic pressure
ω	= eigenfrequency

Introduction

IT is well known that for some structures it is possible to obtain the exact analytic expression of the transfer matrix that relates the state variables on one side of the structure to the same variables on the other side. The solution of complex geometries made by several members can be approached by assembling the exact transfer functions of each member. Nevertheless, the transfer matrix method presents two kinds of difficulties: first, the deduction of the transfer matrix is not always easy and often possible only for elementary members; second, its practical use is cumbersome, because almost always transcendental equations are involved.

The present approach of analysis is based on an analytical solution of the differential equations that describe the behavior of a single elementary member. The solution is obtained as the series expansion in terms of one parameter to obtain a mathematical model named parameter-transfer finite element (P-TFE). Complex geometries can be modeled by assembling these elements.[1,2] A simple mathematical model of a structural problem, which can be solved by standard library routines, can be obtained. In fact, the series expansion solution gives a polynomial and not a transcendental expression of unknowns. From a numerical point of view, the approximate solution, up to the desired term, can be obtained by manipulating a polynomial matrix.

The present method is very general and can be applied to several physical problems in which the solution involves some parameter. Some cases of interest might be thermal, aeroelastic, and stability problems. For each of these cases, it is possible to create a library of parametric finite elements, which takes into account the behavior of the structure in the whole.

The proposed approach can be particularly useful in those cases in which the exact transfer function is not known and also in which classical approximate approaches, such as the Galerkin or finite element methods (FEM), are not applicable without strong computational efforts.

In the following paragraph the theoretical formulation of the method is given. The method is then validated by an application to the case of monodimensional panel flutter. This problem has been extensively studied by a number of authors using different approaches such as Galerkin's method[3-7] or finite elements.[8-13] Comparisons with the results obtained with the present method are reported in the numerical examples.

Theoretical Formulation

Following the classical procedure of the transfer function method, the relationship between the state variables of interfaces i and $i-1$ can be defined as follows:

$$\{Y\}_i = [R]_i \{Y\}_{i-1} \tag{1}$$

where $[R]_i$ is the transfer matrix of the ith element E_i. The behavior of the element E_i is described by the following system of differential equations

$$\{Y\}_i' = [A(\lambda, \sigma, \gamma, \ldots)]_i \{Y\}_i + \{F\}_i \tag{2}$$

and by the corresponding conditions at the left:

$$\{Y\}_i = \{Y^*\} \tag{3}$$

Received Jan. 15, 1991; revision received March 16, 1992; accepted for publication March 26, 1992.

*Full Professor, Dipartimento Aerospaziale, Via Eudossiana 16.
†Assistant Professor, Dipartimento Aerospaziale, Via Eudossiana 16.
‡Research Associate, Dipartimento Aerospaziale, Via Eudossiana 16.

where λ, σ, γ, ... are parameters related to inertial forces, compressive loads, dynamic pressures, etc., on which the differential equations (2) are dependent. In the next formulas, we can neglect the subscript i.

The proposed approach starts by expanding $[A(\lambda, \sigma, \gamma, \ldots)]$ into power series of λ, σ, γ, ... Expansion into a series of powers of λ yields for the ith $[A]$

$$[A(\lambda, \sigma, \gamma, \ldots)] = \sum_{p=0}^{\infty} \lambda^p [A(\sigma, \gamma, \ldots)]_p$$

At this stage it is possible to expand $[A(\sigma, \gamma, \ldots)]_p$ into a series of σ, obtaining

$$[A(\lambda, \sigma, \gamma, \ldots)] = \sum_{p=0}^{\infty} \sum_{q=0}^{\infty} \lambda^p \sigma^q [A(\gamma, \ldots)]_{pq}$$

The most general expression of $[A]$ will be

$$[A(\lambda, \sigma, \gamma, \ldots)] = \sum_{p=0}^{\infty} \sum_{q=0}^{\infty} \sum_{r=0}^{\infty} \lambda^p \sigma^q \gamma^r \ldots [A(\ldots)]_{pqr\ldots} \tag{4}$$

However, it is not necessary to expand the expression of $[A(\lambda, \sigma, \gamma \ldots)]$ into a series of each of the parameters λ, σ, γ ... In some cases it is better to confine the expansion to one or two parameters; in this way the dependence of $[A]$ on the other parameters will continue to appear in analytic implicit form.

If $[A]$ depends on one parameter only, by expanding the matrix $[A(\lambda)]$ in power series of λ, Eq. (2) can be written for the ith element in the form of

$$\{Y\}' = \sum_{p=0}^{\infty} \lambda^p [A]_p \{Y\} + \{F\} \tag{5}$$

and the solution can be taken as

$$\{Y\} = \sum_{m=0}^{\infty} \lambda^m \{Y\}_m \tag{6}$$

For the principle of identity of series Eq. (5) reads

$$\{Y\}'_m = [A]_0\{Y\}_m + \delta_{0m}\{F\} + (1-\delta_{om})\{G\}_m \qquad m = 0, 1, \ldots \tag{7}$$

where

$$\{G\}_m = \sum_{p=1}^{m} [A]_p \{Y\}_{m-p}$$

Consequently, the solution can be put in the form

$$\{Y\}_m = [N]_m\{C\}_m + \delta_{om}\{f\} + (1-\delta_{om})\{g\}_m \qquad m = 0, 1, \ldots \tag{8}$$

The first term on the right-hand side is the general solution of the homogeneous equation associated with Eq. (7); the other terms are a particular solution of the full equation. The vectors $\{C\}_m$ are the vectors of integration constants.

In analogy with the FEM, the elements of vectors $\{N\}$ can be thought of as the shape functions of the element. In FEM the choice of shape functions is related to the convergence as the mesh is more and more refined. In the present method, the choice of $\{N\}_m$ is related to the convergence of the series expansion (6).

To satisfy the conditions (3), we impose

$$\{Y\}_0 = \{Y^*\}, \qquad \{Y\}_m = 0 \text{ with } m \neq 0 \tag{9}$$

from which it follows

$$\{C\}_0 = [N^*]_0^{-1}(\{Y^*\} - \{f\}) \tag{10}$$

and

$$\{C\}_m = -[N^*]_m^{-1}\{g\}_m \text{ with } m \neq 0 \tag{11}$$

Equation (6) can now be written as follows:

$$\{Y\} = [N]_0[N^*]_0^{-1}\{Y^*\} + ([I] - [N]_0[N^*]_0^{-1})\{f\} + \sum_{m=1}^{\infty} \lambda^m([I] - [N]_m[N^*]_m^{-1}\{g\}_m \tag{12}$$

As soon as $\{Y\}$ is known, we can set up the extended state vector and extended transfer matrix $[R]_i$.

If a complex structure is considered, it is possible to obtain the expression of the transfer matrix $[R]$ as

$$[R] = [R]_N^{M_N} \ldots [R]_2^{M_2} [R]_1^{M_1}$$

where $[R]_i$ is the transfer matrix of the ith element and M_i is the degree of approximation that concerns the ith element.

It is also possible to consider a simple element as a repetitive structure using a discretization technique. In fact, one can divide the elements into N subelements and expand the polynomial expression (6) up to M.

Development of Parameter-Transfer Finite Elements for Elementary Structures

To show that the general formulation developed in the previous section has many possible applications, some cases are examined concerning classical problems of structural engineering. For each case it is possible to develop a specific

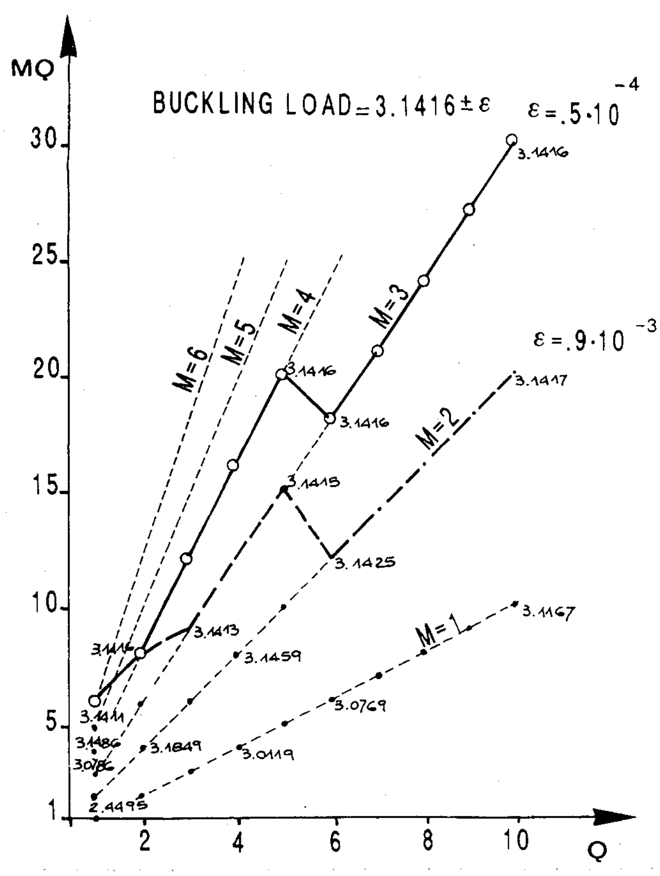

Fig. 1 Convergence of the critical load.

P-TFE in terms of a parameter that accounts for not only the geometric and elastic characteristics of the structure but also the applied action (such as inertia forces, aerodynamic pressure, and compressive load). The expressions of some P-TFE that account for one or more than one applied action are reported.

Dynamic Behavior of an Eulerian Beam (the Dynamic-TFE)

In the case of free vibrations of a beam, Eq. (5) can be written as

$$\begin{bmatrix} w \\ \vartheta \\ M \\ T \end{bmatrix}' = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} w \\ \vartheta \\ M \\ T \end{bmatrix} + \lambda \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} w \\ \vartheta \\ M \\ T \end{bmatrix}$$

It has been assumed $M = Ml/EI$, $T = Tl^2/EI$, $\lambda = \mu\omega^2 l^4/EI$. The expression for the transfer function matrix reads

$$[R] = \sum_{p=0}^{P} \lambda^p [R_{ij}]$$

where

$$R_{11} = R_{22} = R_{33} = R_{44} = \frac{(x-x_0)^{4p}}{(4p)!}$$

$$R_{21} = R_{32} = R_{43} = (1-\delta_{0p})\frac{(x-x_0)^{4p-1}}{(4p-1)!}$$

$$R_{12} = R_{23} = R_{34} = \frac{(x-x_0)^{4p+1}}{(4p+1)!}$$

$$R_{13} = R_{24} = (1-\delta_{0p})\frac{(x-x_0)^{4p+2}}{(4p+2)!}$$

$$R_{31} = R_{42} = (1-\delta_{0p})\frac{(x-x_0)^{4p-2}}{(4p-2)!}$$

$$R_{41} = (1-\delta_{0p})\frac{(x-x_0)^{4p-3}}{(4p-3)!}$$

$$R_{14} = \frac{(x-x_0)^{4p+3}}{(4p+3)!}$$

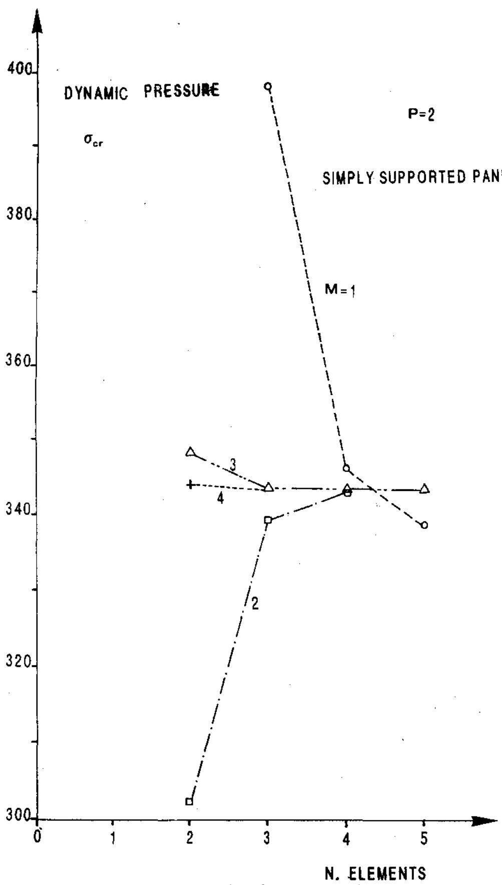

Fig. 2 Convergence of the critical pressure with respect to the number of elements and the order of series expansion.

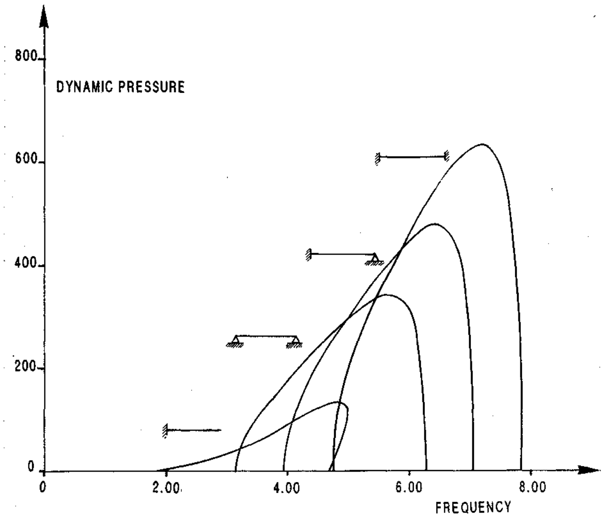

Fig. 3 Coalescence of the first two modes for different boundary conditions.

Buckling of a Beam (the Buckling-TFE)

For the case of the Eulerian buckling load, Eq. (5) can be written as

$$\begin{bmatrix} w \\ \vartheta \\ M \\ T \end{bmatrix}' = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} w \\ \vartheta \\ M \\ T \end{bmatrix} - \gamma \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} w \\ \vartheta \\ M \\ T \end{bmatrix}$$

In this case $\gamma = Pl^2/EI$ where P is the applied load. The expression for $[R]$ is as follows:

$$[R] = \sum_{p=0}^{P} (-1)^p \gamma^p [R_{ij}]$$

where

$$R_{11} = R_{44} = \delta_{0p}$$

$$R_{14} = \frac{(x-x_0)^{2p+3}}{(2p+3)!}$$

$$R_{12} = R_{23} = R_{34} = \frac{(x-x_0)^{2p+1}}{(2p+1)!}$$

$$R_{13} = R_{24} = \frac{(x-x_0)^{2p+2}}{(2p+2)!}$$

$$R_{22} = R_{33} = \frac{(x-x_0)^{2p}}{(2p)!}$$

$$R_{32} = (1-\delta_{0p})\frac{(x-x_0)^{2p} - 1}{(2p-1)!}$$

$$R_{21} = R_{31} = R_{41} = R_{42} = R_{43} = 0$$

Monodimensional Panel Flutter (the Flutter-TFE)

Consider the small oscillation of a flat plate of infinite length, taking into account aerodynamic terms in accordance with the linearized formula of the "piston theory." The set of equations reads

$$\begin{bmatrix} w \\ \vartheta \\ M \\ T \end{bmatrix}' = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -\gamma & 0 & 1 \\ \lambda & -\sigma & 0 & 0 \end{bmatrix} \begin{bmatrix} w \\ \vartheta \\ M \\ T \end{bmatrix} \tag{13}$$

It has been assumed $M = Ml/EI$, $T = Tl^2/EI$, $\gamma = Pl^2/EI$, $\sigma = 2ql^3/EI\beta$, and $\lambda = \mu\omega^2 l^4/EI$, where ω is a complex number. The matrix A is in the form $A = A(\lambda, \gamma, \sigma)$. We can start by assuming

$$[A(\lambda, \sigma, \gamma)] = [A_0(\sigma, \gamma)] + \lambda[A_1] \tag{14}$$

If we assume

$$\{Y(\lambda, \sigma, \gamma)\} = \sum_{p=0}^{\infty} \lambda^p \{Y(\sigma, \gamma)\}_p \tag{15}$$

then Eq. (13) is transformed, yielding

$$\sum_{p=0}^{\infty} \lambda^p \{Y(\sigma, \gamma)\}_p' = [A_0(\sigma, \gamma)] \sum_{p=0}^{\infty} \lambda^p \{Y(\sigma, \gamma)\}_p + \lambda[A_1] \sum_{p=0}^{\infty} \lambda^p \{Y(\sigma, \gamma)\}_p \tag{16}$$

It is possible to order the terms of Eq. (16) with respect to the power of λ; Eq. (16) can be considered equivalent to the following set of equations:

$$\{Y(\sigma, \gamma)\}_p' = [A_0(\sigma, \gamma)]\{Y(\sigma, \gamma)\}_p + (1 - \delta_{0p})[A_1]\{Y(\sigma, \gamma)\}_{p-1} \qquad p = 0, 1 \ldots \tag{17}$$

and initial conditions according to Eqs. (3).

At this stage it is possible to proceed two distinct ways 1) by solving analytically the set of Eq. (17), 2) by considering each equation of the set of Eq. (17) as the type (2). In the first case, the λ-TFE will be developed, that is, the element that accounts for the dynamic behavior, in which the terms still depend from σ and γ. In the second case, it will be assumed that the unknown depends on one of the remaining parameters as in a power series expansion. A $\lambda\sigma$-TFE or a $\lambda\gamma$-TFE will be developed.

To obtain a $\lambda\sigma$-TFE, we assume

$$\{Y(\sigma, \gamma)\}_p = \sum_{m=0}^{\infty} \sigma^m \{Y(\gamma)\}_{pm} \tag{18}$$

and

$$[A_0(\sigma, \gamma)] = [A_{00}(\gamma)] + \sigma[A_{01}] \tag{19}$$

Analogous to what was done for Eq. (16) the following set of equations is obtained:

$$\{Y(\gamma)\}_{pm}' = [A_{00}(\gamma)]\{Y(\gamma)\}_{pm} + (1 - \delta_{om})[A_{01}]\{Y(\gamma)\}_{p(m-1)} + (1 - \delta_{op})[A_1]\{Y(\gamma)\}_{(p-1)m}$$

$$\text{with } p = 0, 1 \ldots, m = 0, 1 \ldots \tag{20}$$

and initial conditions according to Eqs. (3).

Note that Eqs. (20) are dependent on only one parameter. We can once more make an assumption about the unknowns as follows:

$$\{Y(\gamma)\}_{pm} = \sum_{n=0}^{\infty} \gamma^n \{Y\}_{pmn} \tag{21}$$

with

$$[A_{00}(\gamma)] = [A_{000}] + \gamma[A_{001}]$$

The derived equations will not depend on any parameter and are very easy to solve:

$$\{Y\}_{pmn}' = [A_{000}]\{Y\}_{pmn} + (1 - \delta_{0n})[A_{001}]\{Y\}_{pm(n-1)} + (1 - \delta_{0m})[A_{01}]\{Y\}_{p(m-1)n} + (1 - \delta_{0p})[A_1]\{Y\}_{(p-1)mn}$$

$$\text{with } p = 0, 1 \ldots, m = 0, 1 \ldots, n = 0, 1 \ldots \tag{22}$$

Having obtained the solutions of Eq. (22), it is possible to express the original unknown as follows:

$$\{Y\} = \sum_{p=0}^{\infty} \lambda^p \sum_{m=0}^{\infty} \sigma^m \sum_{n=0}^{\infty} \gamma^n \{Y\}_{pmn} \tag{23}$$

where $\{Y\}_{pmn}$ are known functions. By stopping the series expansion, respectively, at $p = P$, $m = M$, and $n = N$, it is possible to get an approximate solution of the panel flutter problem, because the unknown is now expressed by means of a polynomial expression of the parameters.

Table 1 Critical pressure σ and corresponding frequency λ for different kinds of supports

	Hinged-hinged	Clamped-clamped	Clamped-hinged	Clamped-free
Present results, σ	343.356	636.581	479.532	135.301
Weisshaar, [7] σ	343.2			
Mei, [12] σ	343.280	636.586		
Exact result,[12] σ	343.3564	636.5691		
Present results, λ	5.69	7.24	6.46	4.84
Mei,[12] λ	5.69	7.24	6.46	
Exact result,[12] λ	5.69	7.24	6.46	

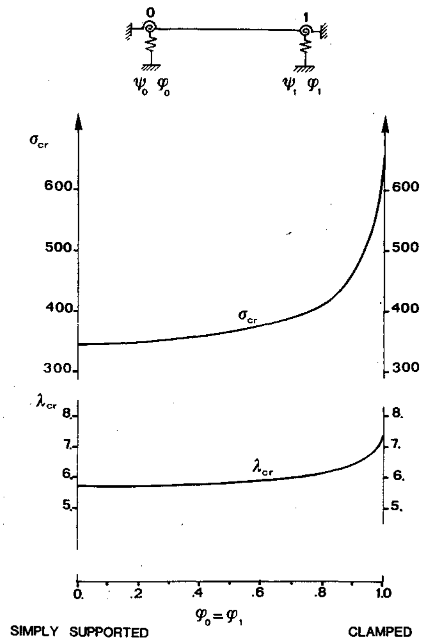

Fig. 4 Effect of the stiffness of the constraint on the values of critical pressure and critical frequency of flutter [for $x = 0$: $M = a\vartheta$; for $x = l$: $M = -a\vartheta$ where a is a constant varying from 0 (simply supported panel) to ∞ (clamped panel); for sake of simplicity in the diagrams of the values of σ and λ are reported vs $\varphi = a/(1-a)$].

The numerical procedure can be improved by considering the beam under concern divided into substructures and by applying to each subelement the approximate procedure just presented. In this way the accuracy of the procedure can be improved both by increasing the order of the expansions and by refining the P-TFE mesh; in fact, the original panel can be subdivided into elements and each element can be treated as mentioned before.

The expression for the final transfer function matrix that is for the $\lambda\sigma\gamma$-TFE reads

$$[R]=\sum_{p=0}^{P}\lambda^p\sum_{m=0}^{M}(-1)^m\sigma^m\sum_{n=0}^{N}(-1)^n\gamma^n[R_{ij}]_{pmn}$$

where

$$[R_{ij}]_{pmn}=[S_{ij}]_{pmn}\,[K_{ij}]_{pmn}$$

If we give for $[K]_{100}$, $[K]_{010}$, $[K]_{001}$, and $[K]_{000}$ the following expressions

$$[K]_{000}=[K]_{100}=[I],\qquad [K]_{001}=\begin{bmatrix}0&0&0&0\\0&0&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$$

$$[K]_{010}=\begin{bmatrix}0&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$$

$[K]_{pmn}$ can be derived as follows:

$$[K]_{pmn}=(1-\delta_{op})[K]_{(p-1)mn}+(1-\delta_{om})[K]_{p(m-1)n}+(1-\delta_{on})[K]_{pm(n-1)}$$

As far as $[S_{ij}]_{pmn}$ is concerned we obtain:

$$S_{11}=S_{22}=S_{33}=S_{44}=\frac{(x-x_0)^{2n+3m+4p}}{(2n+3m+4p)!}$$

$$S_{21}=S_{32}=S_{43}=C\frac{(x-x_0)^{2n+3m+4p-1}}{(2n+3m+4p-1)!}$$

$$S_{12}=S_{23}=S_{34}=\frac{(x-x_0)^{2n+3m+4p+1}}{(2n+3m+4p+1)!}$$

$$S_{31}=S_{42}=C\frac{(x-x_0)^{2n+3m+4p-2}}{(2n+3m+4p-2)!}$$

$$S_{13}=S_{24}=\frac{(x-x_0)^{2n+3m+4p+2}}{(2n+3m+4p+2)!}$$

$$S_{14}=\frac{(x-x_0)^{2n+3m+4p+3}}{(2n+3m+4p+3)!}$$

$$S_{41}=C\frac{(x-x_0)^{2n+3m+4p-3}}{(2n+3m+4p-3)!}$$

where

$$C=1-\delta_{0p}\,\delta_{0n}\delta_{0m}$$

The first step of the procedure by which the above expressions were obtained is described in the Appendix.

Convergence of the Method

Convergence can be obtained either by increasing up to infinity the number of terms of the series expansion or by increasing the number of elements into which the structure is subdivided. From a practical point of view, only a finite number of terms of the expression and a finite number of elements can be taken. Consequently it is worthwhile to analyze the rate of convergence using one of the different ways just mentioned.

As far as the dynamic and the buckling-TFE are concerned, in Ref. 1 an extensive analysis of the convergence has been carried out.

Figure 1 shows, for the buckling case, the rate of convergence as a function of the number M of terms of the series expansion and the number Q of elements into which the structure is subdivided. In any case the convergence is assured by assuming at least $P=2$ for the dynamic case and at least $M=3$ for the buckling case.

As far as the aeroelastic case is concerned, the diagrams of Fig. 2, in which also the exact solution is reported,[12] show that even with few elements, a strong rate of convergence is obtained by considering three terms of the series expansion ($M=3$).

For this reason the numerical expression reported in the following were performed assuming the $\lambda\sigma\gamma$-TFE at the following orders of expression, $P=2$, $M=3$, and $N=3$, and using only a very limited number of elements, namely four. These examples, obtained with such a limited number of the terms of series expansion and of the considered elements, show a good agreement with what other authors obtained.

An order of magnitude of computing times can be estimated, considering that each one of the diagrams of Fig. 3 was obtained using a PC 386 in a time on the order of a few minutes.

Numerical Examples

In the first numerical case, it has been assumed that no compressive load is applied, that is, $\gamma=0$. The coalescence of the first two modes as obtained by the present approach is shown in Fig. 3 in which several boundary conditions are

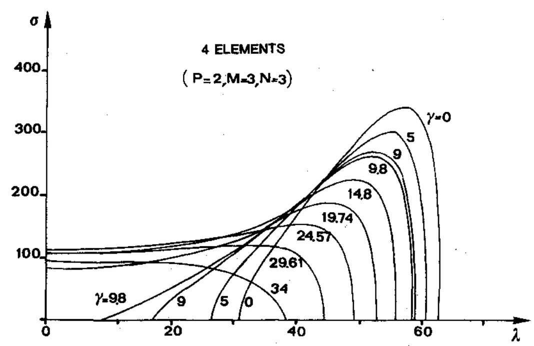

Fig. 5 Effect of the presence of axial load on the values of critical pressure and critical frequency of flutter.

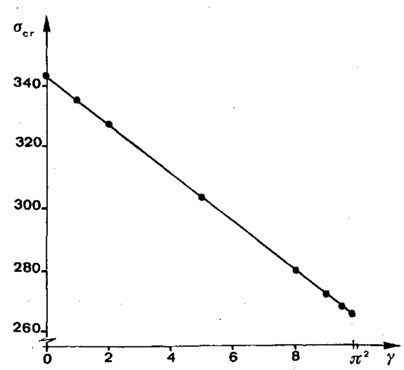

Fig. 6 Critical pressure vs axial load.

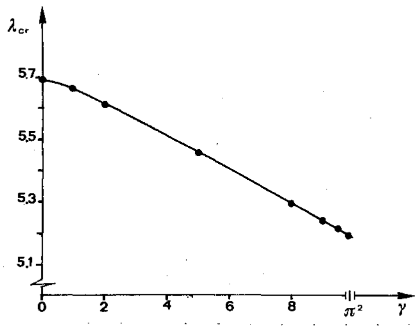

Fig. 7 Critical frequency vs axial load.

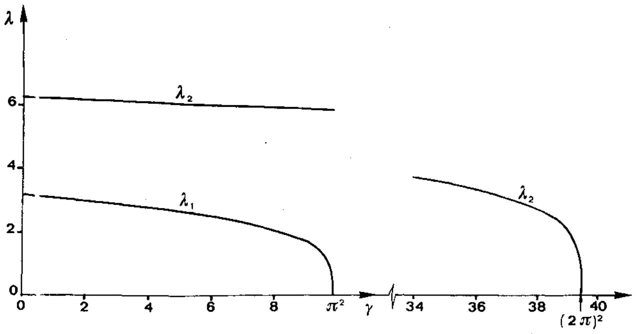

Fig. 8 Variation of frequency vs axial load ($\sigma = 0$).

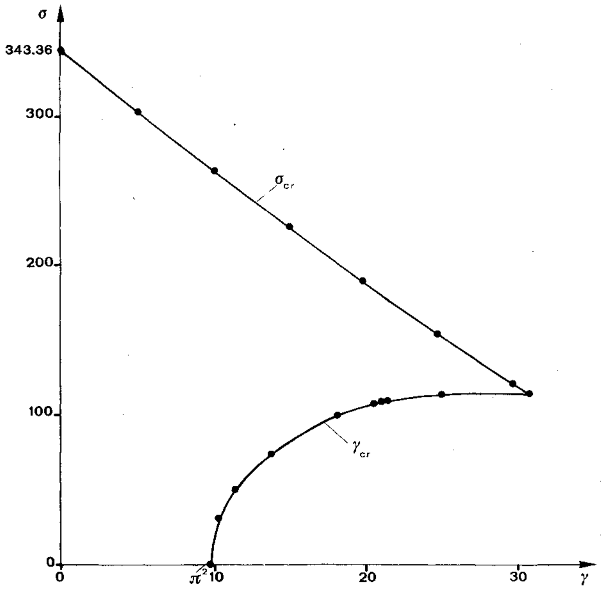

Fig. 9 Combined effect of the presence of dynamic pressure and in-plane axial load on the stability of the panel.

considered. To compare present results with those obtained by other authors, the values of critical pressure are reported in Table 1. The effect of the type of boundary condition on the critical values of dynamic pressure can be observed in Fig. 4 in which the critical pressure and the correspondent critical frequency are reported vs the elastic coefficient of the support at boundary edges.

In Fig. 5 the effect of axial load γ on the critical dynamic pressure can be observed for a simply supported panel. The variation of the critical pressure and of critical frequency vs γ are reported in Figs. 6 and 7. The simple case for which $\sigma = 0$ is also studied for an assessment of the procedure; the variation of the first two eigenfrequencies vs the axial load is reported on Fig. 8.

Figure 9 shows the combined effect of the dynamic pressure and axial load on the aeroelastic stability of the plate. The figure shows an area of stability that is bounded by a curved line that represents the instability due to buckling and another line that represents the instability due to the coalescence of the first two frequencies.

Conclusion

In the previous paragraphs an original method to analyze some classical structural problem was presented. For each case, a P-TFE is developed. The method can be applied without difficulties to more complex cases; it can be implemented very easily and does not need high computing times. The values obtained for the problem of panel flutter show an excellent agreement with well-known solutions.

Appendix

The solution of the first of Eqs. (22) can be obtained easily, considering that the solution is of the form

$$\{Y\}_{000} = \begin{bmatrix} x^3/3! & x^2/2 & x & 1 \\ x^2/2 & x & 1 & 0 \\ x & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix}$$

where C_i are to be determined by imposing the initial conditions $\{Y\}_{000} = \{w^* \ \vartheta^* \ M^* \ T^*\}^T$ at point x_0. By assuming $x_0 = 0$, the solution can be obtained in the following form:

$$\{Y\}_{000} = \begin{bmatrix} 0 & x & x^2/2 & x^3/3! \\ 0 & 1 & x & x^2/2 \\ 0 & 0 & 1 & x \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} w^* \\ \vartheta^* \\ M^* \\ T^* \end{bmatrix}$$

Now it is easy to solve the second of Eqs. (22); the solution of the associated homogeneous equation is the same as the first equation and the boundary conditions are $\{Y\}_{001} = \{0\,0\,0\,0\}^T$. The expression for $\{Y\}_{001}$ reads

$$\{Y\}_{001} = -\begin{bmatrix} 0 & x^3/3! & x^4/4! & x^5/5! \\ 0 & x^2/2 & x^3/3! & x^4/4! \\ 0 & x & x^2/2 & x^3/3! \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} w^* \\ \vartheta^* \\ M^* \\ T^* \end{bmatrix}$$

The expression for $\{Y\}_{00n}$ can be easily derived. It corresponds to the case of the presence of the Eulerian buckling load only, which is reported in the corresponding paragraph. At this stage it is possible to solve Eq. (20) and then Eq. (17), yielding the general solution described in the text.

References

[1]Barboni, R., Castellani, A., and Mannini, A., "Un metodo analitico-numerico per l'analisi strutturale," Atti del Dipartimento Aerospaziale, Università di Roma La Sapienza, DAURS 88.02, Nov. 1988 (in Italian).

[2]Barboni, R., Gaudenzi, P., Mannini, A., and Santini, P., "An Application to the Dynamics on Large Space Structures of a New Semi-Analytic Method for Structural Analysis," International Council of the Aeronautical Sciences, ICAS 90-1.6.4, Sept. 1990.

[3]Santini, P., and Barboni, R., "Contributo allo studio del panel flutter," Atti del Centro Ricerche Aerospaziali, Rep. No. 14, Rome, Dec. 1966 (in Italian).

[4]Dowell, E. H., "Nonlinear Flutter of Curved Plates," *AIAA Journal*, Vol. 7, No. 3, 1969, pp. 424–431.

[5]Eastep, F. E., and McIntosh, S. C., Jr., "Analysis of Nonlinear Panel Flutter and Response Under Random Excitation or Nonlinear Aerodynamic Loading," *AIAA Journal*, Vol. 9, No. 3, 1971, pp. 411-418.

[6]Morino, L., Kwo, C. C., and Dugundji, J., "Perturbation and Harmonic Balance Method for Nonlinear Panel Flutter," *AIAA Journal*, Vol. 10, No. 11, 1972, pp. 1479-1484.

[7]Weisshaar, T. A., "Aeroelastic Optimization for a Panel in High Mach Number Supersonic Flow," *Journal of Aircraft*, Vol. 9, No. 9, 1972, pp. 611-617.

[8]Olson, M. D., "Finite Elements Applied to Panel Flutter," *AIAA Journal*, Vol. 5, No. 12, 1967, pp. 2267-2270.

[9]Yang, T. Y., and Sung, S. H., "Finite Element Panel Flutter in Three-Dimensional Supersonic Unsteady Potential Flow," *AIAA Journal*, Vol. 15, No. 12, 1977, pp. 1677-1683.

[10]Sarama, B. S., and Varadan, T. K., "Nonlinear Panel Flutter by Finite Element Method," *AIAA Journal*, Vol. 26, No. 5, 1988, pp. 566-574.

[11]Gray, C. E., Mei, C., and Shore, C. P., "Finite Element Method for Large Amplitude Two-Dimensional Panel Flutter at Hypersonic Speeds," *AIAA Journal*, Vol. 29, No. 2, 1991, pp. 290-298.

[12]Mei, C., "A Finite-Element Approach for Nonlinear Panel Flutter," *AIAA Journal*, Vol. 15, No. 8, 1977, pp. 1107-1110.

[13]Sander, G., Bon, C., and Geradin, M., "Finite Element Analysis of Supersonic Panel Flutter," *International Journal for Numerical Methods in Engineering*, Vol. 7, No. 3, 1973, pp. 379-394.